复空间形式的紧致全实伪脐子流形

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摘要: 研究了复空间形式中具有平行法平均曲率向量的紧致全实伪脐子流形. 给出了 Ricci 曲率成为全脐子流形的判定条件.

关键词: 复空间形式; 紧致子流形; 全实; 伪脐; 平行法平均曲率向量
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Compact Totally Real Pseudo-Umbilical Submanifolds of Complex Space Forms

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Abstract: The compact totally real pseudoumbilical submanifolds in complex spacec forms are studied. The condition for Ricci curvature to be totally umbilical submanifolds is presented.

Key words: complex space forms; compact submanifolds; totally real; pseudoumbilical; parallel normalized mean curvature vector

Kaehler 流形 M 的子流形 M, 如果映射到其法空间, 则称 M 为全实子流形. 具有常全纯截面曲率的 Kaehler 流形被称为复空间形式. 设 M 是 n 维黎曼流形, M 是 $2(n+p)$ 维 Kaehler 流形, $p \geqslant 0$. 设 J 是 M 的近复结构, g 和 g 分别是 M 和 M 的黎曼度量. ∇ 和 ∇ 分别表示 M 和 M 的共变微分算子. M 在 M 中的第 2 基本形式 h 由下式给出

$$h(X,Y) = \nabla_X Y - \nabla_X Y,$$

这里 X, Y 是 M 的切向量场. 设 ξ 是平均曲率向量, 如果存在 M 上的函数 λ 满足

$$g(h(X,Y),\xi) = \lambda g(X,Y), \qquad (1)$$

则 M 是伪脐的. 如果 $h(X,Y) = g(X,Y)\xi$, 其中 X, Y 为 M 的任意切向量场, 则 M 是全脐的. 法平均曲率向量是指平行于平均曲率向量的单位向量. 本文研究了伪脐子流形, 得到如下结果:

定理 1　设 M^n 是 $M^{n+p}(\tilde{c})$ ($\tilde{c} \leqslant 0$) 中具有非零平行法平均曲率向量的 n 维紧致全实伪脐子流形, 如果 M^n 的 Ricci 曲率 Q 满足条件:

$$Q \geqslant \frac{1}{4}(n-2)(\tilde{c}+4H^2) - \frac{1}{4n}\tilde{c},$$

其中 $Q(x)$ 为 M 在其点 x 的 Ricci 曲率的下确界, 则 M^n 是全脐的.

1　基本公式

M 与 M 上的结构和度量如前所示, 在 M 上选取标准正交局部标架场: $e_1, \cdots, e_n, e_{n+1}, \cdots, e_{n+p}, e_{1^*} = Je_1, \cdots, e_{n^*} = Je_n, e_{(n+1)^*} = Je_{n+1}, \cdots, e_{(n+p)^*} = Je_{n+p}$. 其对偶标架场为 $\omega^1, \cdots, \omega^n, \omega^{n+1}, \cdots, \omega^{n+p}, \omega^{1^*}, \cdots, \omega^{n^*}, \omega^{(n+1)^*}, \cdots, \omega^{(n+p)^*}$, 则 M 的结构方程由下式给出

$$d\omega^A = -\sum \omega_B^A \wedge \omega^B, \omega_B^A + \omega_A^B = 0, \qquad (2)$$

$$R_{jkl}^i = \tilde{R}_{jkl}^i + \sum (h_{ik}^\alpha h_{jl}^\alpha - h_{il}^\alpha h_{jk}^\alpha), \qquad (3)$$

$$\omega_i^\alpha = \sum h_{ij}^\alpha \omega^j, h_{ij}^\alpha = h_{ji}^\alpha = g(A_\alpha e_i, e_j). \qquad (4)$$

h_{ij}^α 的共变导数 h_{ijk}^α, h_{ijkl}^α 及第 2 基本形式 h_{ij}^α 的 LaplacianΔh_{ij}^α 详细见文献[1-2].

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$$\sum_k h^\alpha_{ijk}\omega^k = dh^\alpha_{ij} - \sum_l h^\alpha_{il}\omega^l_j - \sum_l h^\alpha_{lj}\omega^l_i + \sum_\beta h^\beta_{ij}\omega^\alpha_\beta, \tag{5}$$

$$\sum_l h^\alpha_{ijkl}\omega^l = dh^\alpha_{ijk} - \sum_l h^\alpha_{ljk}\omega^l_i - \sum_l h^\alpha_{ilk}\omega^l_j - \sum_l h^\alpha_{ijl}\omega^l_k + \sum_\beta h^\beta_{ijk}\omega^\alpha_\beta, \tag{6}$$

$$\Delta h^\alpha_{ij} = \sum_k h^\alpha_{ijkk} = \sum_k (h^\alpha_{kkij} - \tilde{R}^\alpha_{kikj} - \tilde{R}^\alpha_{ijkk}) + \sum_k (\sum_m h^\alpha_{km}R^m_{ijk} + \sum_m h^\alpha_{mi}R^m_{kjk} - \sum_\beta h^\beta_{ki}R^\alpha_{\beta jk}). \tag{7}$$

$M^{n+p}(\tilde{c})$ 的曲率张量 $\tilde{R}^A_{BCD}$ 的共变导数为 $\tilde{R}^A_{BCD;E}$，限制在 M 上为 $\tilde{R}^\alpha_{ijk;l}$ [2].

$$\tilde{R}^\alpha_{ijk;l} = \tilde{R}^\alpha_{ijkl} - \sum \tilde{R}^\alpha_{\beta jk}h^\beta_{il} - \sum \tilde{R}^\alpha_{i\beta k}h^\beta_{jl} - \sum \tilde{R}^\alpha_{ij\beta}h^\beta_{kl} + \sum \tilde{R}^m_{ijk}h^\alpha_{ml}, \tag{8}$$

由于 $M^{n+p}(\tilde{c})$ 是局部对称的,故有 $\tilde{R}^A_{BCD;E}=0$.

对具有常全纯截面曲率 $\tilde{c}$ 的复 $n+p$ 维复空间形式 $M^{n+p}(\tilde{c})$，有 $\tilde{R}^A_{BCD}$ [1]

$$\tilde{R}^A_{BCD} = \frac{1}{4}\tilde{c}(\delta_{AC}\delta_{BD} - \delta_{AD}\delta_{BC} + \tilde{J}_{AC}\tilde{J}_{BD} - \tilde{J}_{AD}\tilde{J}_{BC} + 2\tilde{J}_{AB}\tilde{J}_{CD}). \tag{9}$$

设 M 是 $M^{n+p}(\tilde{c})$ 的一 n 维全实子流形,由式(3)和式(9)有

$$R^i_{jkl} = \frac{1}{4}\tilde{c}(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}) + \sum (h^\alpha_{ik}h^\alpha_{jl} - h^\alpha_{il}h^\alpha_{jk}). \tag{10}$$

设 M 的截面曲率为 K,X、Y 是正交单位向量,有

$$K(X,Y) = \frac{1}{4}\tilde{c} + g(h(X,X),h(Y,Y)) - \|h(X,Y)\|^2. \tag{11}$$

若 M 是伪脐子流形,当 $H\neq 0$ 时,可取 ξ 平行于 e_{n+1},由式(1)有

$$\mathrm{tr}\, H_{n+1} = nH,\ \mathrm{tr}\, H_\alpha = 0, \alpha\neq n+1, \tag{12}$$

由式(12)得 $\sum \mathrm{tr}\, H_\alpha h^\alpha_{ij} = n\lambda\delta_{ij}, H^2 = \lambda$,有

$$h^{n+1}_{ij} = H\delta_{ij}. \tag{13}$$

若 M 还具有平行的法平均曲率向量，则对所有 α 有

$$\omega^\alpha_{n+1} = 0. \tag{14}$$

由式(8)得

$$\tilde{R}^\alpha_{kikj} = \sum \tilde{R}^\alpha_{\beta ik}h^\beta_{kj} + \sum \tilde{R}^\alpha_{k\beta k}h^\beta_{ij} + \sum \tilde{R}^\alpha_{ki\beta}h^\beta_{kj} - \sum \tilde{R}^m_{kik}h^\alpha_{mj}. \tag{15}$$

$$\tilde{R}^\alpha_{ijkk} = \sum \tilde{R}^\alpha_{\beta jk}h^\beta_{ik} + \sum \tilde{R}^\alpha_{i\beta k}h^\beta_{jk} + \sum \tilde{R}^\alpha_{ij\beta}h^\beta_{kk} - \sum \tilde{R}^m_{ijk}h^\alpha_{mk}. \tag{16}$$

由式(7)得

$$\frac{1}{2}\Delta(\sum_{i,j,\alpha}(h^\alpha_{ij})^2) = \sum_{i,j,k,\alpha}(h^\alpha_{ijk})^2 + \sum_{i,j,\alpha}h^\alpha_{ij}\Delta h^\alpha_{ij} = \sum_{i,j,k,\alpha}(h^\alpha_{ijk})^2 + \sum_{i,j,\alpha}h^\alpha_{ij}\sum_k(h^\alpha_{kkij} - \tilde{R}^\alpha_{kikj} - \tilde{R}^\alpha_{ijkk}) + \sum_{i,j,\alpha}h^\alpha_{ij}\sum_k(\sum_m h^\alpha_{km}R^m_{ijk} + \sum_m h^\alpha_{mi}R^m_{kjk} - \sum_\beta h^\beta_{ki}R^\alpha_{\beta jk}). \tag{17}$$

由式(4)、(5)、(12)和(14),对于 $\alpha\neq n+1$,有

$$\sum_{i,k}h^\alpha_{iik}\omega^k = d\sum_i h^\alpha_{ii} - \sum_{i,l}h^\alpha_{il}\omega^l_i - \sum_{i,l}h^\alpha_{li}\omega^l_i + \sum_{i,\beta}h^\beta_{ii}\omega^\alpha_\beta = nH\omega^\alpha_{n+1} = 0. \tag{18}$$

由式(4)、(5)、(14)和(18)，对于 $\alpha\neq n+1$，有

$$\sum_{\substack{k,j\\ \alpha\neq n+1}}h^\alpha_{kkij}\omega^j = d\sum_{\substack{k\\ \alpha\neq n+1}}h^\alpha_{kki} - \sum_{\substack{k,j\\ \alpha\neq n+1}}h^\alpha_{jki}\omega^j_k - \sum_{\substack{k,j\\ \alpha\neq n+1}}h^\alpha_{kji}\omega^j_k - \sum_{\substack{k,j\\ \alpha\neq n+1}}h^\alpha_{kkj}\omega^j_i + \sum_{\substack{k,\beta\\ \alpha\neq n+1}}h^\beta_{kki}\omega^\alpha_\beta = \sum_{\substack{k\\ \alpha\neq n+1}}h^{n+1}_{kki}\omega^\alpha_{n+1} = 0. \tag{19}$$

把式(9)代入式(15)和(16),计算得

$$\sum_{\substack{i,j,k,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}\tilde{R}^\alpha_{kikj} = 0, \sum_{\substack{i,j,k,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}\tilde{R}^\alpha_{ijkk} = 0. \tag{20}$$

由式(19)和(20),对于 $\alpha\neq n+1$,式(17)为

$$\frac{1}{2}\Delta(\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h^\alpha_{ij})^2) = \sum_{\substack{i,j,k,\alpha\\ \alpha\neq n+1}}(h^\alpha_{ijk})^2 + \sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}\sum_k(\sum_m h^\alpha_{km}R^m_{ijk} + \sum_m h^\alpha_{mi}R^m_{kjk} - \sum_\beta h^\beta_{ki}R^\alpha_{\beta jk}), \tag{21}$$

下面化简式(21)，如果用 $\boldsymbol{H}_\alpha$ 表示对称矩阵(h^α_{ij}),由式(9)、(10)及(13),对于 $\alpha\neq n+1$,

$$\sum_{\substack{i,j,k,m,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}h^\alpha_{km}R^m_{ijk} + \sum_{\substack{i,j,k,m,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}h^\alpha_{mi}R^m_{kjk} = \sum_{\substack{i,j,k,m,\beta,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}h^\alpha_{km}h^\beta_{mj}h^\beta_{ik} - \sum_{\substack{i,j,k,m,\beta,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}h^\alpha_{km}h^\beta_{mk}h^\beta_{ij} + \sum_{\substack{i,j,k,m,\beta,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}h^\alpha_{mi}h^\beta_{mj}h^\beta_{kk} - \sum_{\substack{i,j,k,m,\beta,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}h^\alpha_{mi}h^\beta_{mk}h^\beta_{kj} + \frac{1}{4}n\tilde{c}\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h^\alpha_{ij})^2 = \sum_{\substack{\beta,\alpha\\ \alpha\neq n+1}}\mathrm{tr}\,(\boldsymbol{H}_\alpha\boldsymbol{H}_\beta)^2 - \sum_{\substack{\beta,\alpha\\ \alpha\neq n+1}}(\mathrm{tr}\,\boldsymbol{H}_\alpha\boldsymbol{H}_\beta)^2 + nH^2\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h^\alpha_{ij})^2 - \sum_{\substack{\beta,\alpha\\ \alpha\neq n+1}}\mathrm{tr}(\boldsymbol{H}^2_\alpha\boldsymbol{H}^2_\beta) + \frac{1}{4}n\tilde{c}\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h^\alpha_{ij})^2. \tag{22}$$

由式(13)得

$$\sum_{\alpha\neq n+1}\mathrm{tr}\,(\boldsymbol{H}_\alpha\boldsymbol{H}_{n+1})^2 - \sum_{\alpha\neq n+1}\mathrm{tr}(\boldsymbol{H}^2_\alpha\boldsymbol{H}^2_{n+1}) = 0. \tag{23}$$

式(22)变为

$$\sum_{\substack{i,j,k,m,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}h^\alpha_{km}R^m_{ijk} + \sum_{\substack{i,j,k,m,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}h^\alpha_{mi}R^m_{kjk} = \frac{1}{2}\sum_{\beta,\alpha\neq n+1}\mathrm{tr}(\boldsymbol{H}_\alpha\boldsymbol{H}_\beta - \boldsymbol{H}_\beta\boldsymbol{H}_\alpha)^2 - \sum_{\beta,\alpha\neq n+1}(\mathrm{tr}\,\boldsymbol{H}_\alpha\boldsymbol{H}_\beta)^2 + \frac{1}{4}n\tilde{c}\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h^\alpha_{ij})^2 + nH^2\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h^\alpha_{ij})^2. \tag{24}$$

$$\sum_{\substack{i,j,k,\beta,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}h^\beta_{ki}R^\alpha_{\beta jk} = \sum_{\substack{i,j,k,l,\beta,\alpha\\ \alpha\neq n+1}}h^\alpha_{ij}h^\beta_{ki}h^\alpha_{lj}h^\beta_{lk} -$$

$$\sum_{\substack{i,j,k,l,\beta,\alpha\\ \alpha\neq n+1}} h_{ij}^{\alpha}h_{ki}^{\beta}h_{lk}^{\alpha}h_{lj}^{\beta}+\frac{1}{4}\tilde{c}\sum_{\substack{i,j,k,\beta,\alpha\\ \alpha\neq n+1}} h_{ij}^{\alpha}h_{ki}^{\beta}(\delta_{\alpha j^*}\delta_{\beta k^*}-\delta_{\alpha k^*}\delta_{\beta j^*})=\sum_{\substack{\beta,\alpha\\ \alpha\neq n+1}}\mathrm{tr}(\boldsymbol{H}_{\alpha}^2\boldsymbol{H}_{\beta}^2)-\sum_{\substack{\beta,\alpha\\ \alpha\neq n+1}}\mathrm{tr}(\boldsymbol{H}_{\alpha}\boldsymbol{H}_{\beta})^2-$$

$$\frac{1}{4}\tilde{c}\sum_{\substack{i,j\\ k^*\neq n+1}}(h_{ij}^{k^*})^2+\frac{1}{4}\tilde{c}\sum_{\substack{i,j,k\\ j^*\neq n+1}}h_{ij}^{j^*}h_{ki}^{k^*}=$$

$$\sum_{\beta,\alpha\neq n+1}\mathrm{tr}(\boldsymbol{H}_{\alpha}^2\boldsymbol{H}_{\beta}^2)-\sum_{\beta,\alpha\neq n+1}\mathrm{tr}(\boldsymbol{H}_{\alpha}\boldsymbol{H}_{\beta})^2+\sum_{\alpha\neq n+1}\mathrm{tr}(\boldsymbol{H}_{\alpha}^2\boldsymbol{H}_{n+1}^2)-\sum_{\alpha\neq n+1}\mathrm{tr}(\boldsymbol{H}_{\alpha}\boldsymbol{H}_{n+1})^2-$$

$$\frac{1}{4}\tilde{c}\sum_{\substack{i,j\\ k^*\neq n+1}}(h_{ij}^{k^*})^2+\frac{1}{4}\tilde{c}\sum_{\substack{i,j,k\\ j^*\neq n+1}}h_{ij}^{j^*}h_{ki}^{k^*}=$$

$$\sum_{\beta,\alpha\neq n+1}\mathrm{tr}(\boldsymbol{H}_{\alpha}^2\boldsymbol{H}_{\beta}^2)-\sum_{\beta,\alpha\neq n+1}\mathrm{tr}(\boldsymbol{H}_{\alpha}\boldsymbol{H}_{\beta})^2-\frac{1}{4}\tilde{c}\sum_{\substack{i,j\\ k^*\neq n+1}}(h_{ij}^{k^*})^2+\frac{1}{4}\tilde{c}\sum_{\substack{i,j,k\\ j^*\neq n+1}}h_{ij}^{j^*}h_{ki}^{k^*}=$$

$$-\frac{1}{2}\sum_{\beta,\alpha\neq n+1}\mathrm{tr}(\boldsymbol{H}_{\alpha}\boldsymbol{H}_{\beta}-\boldsymbol{H}_{\beta}\boldsymbol{H}_{\alpha})^2-\frac{1}{4}\tilde{c}\sum_{\substack{i,j\\ k^*\neq n+1}}(h_{ij}^{k^*})^2+\frac{1}{4}\tilde{c}\sum_{\substack{i,j,k\\ j^*\neq n+1}}h_{ij}^{j^*}h_{ki}^{k^*}. \qquad (25)$$

2 定理证明

定理1的证明:式(24)、(25)代入式(21)得

$$\frac{1}{2}\Delta\Big(\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h_{ij}^{\alpha})^2\Big)=\sum_{\substack{i,j,k,\alpha\\ \alpha\neq n+1}}(h_{ijk}^{\alpha})^2+\sum_{\beta,\alpha\neq n+1}\mathrm{tr}(\boldsymbol{H}_{\alpha}\boldsymbol{H}_{\beta}-\boldsymbol{H}_{\beta}\boldsymbol{H}_{\alpha})^2-\sum_{\beta,\alpha\neq n+1}(\mathrm{tr}\,\boldsymbol{H}_{\alpha}\boldsymbol{H}_{\beta})^2+$$

$$\frac{1}{4}n\tilde{c}\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h_{ij}^{\alpha})^2+nH^2\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h_{ij}^{\alpha})^2+\frac{1}{4}\tilde{c}\sum_{\substack{i,j\\ k^*\neq n+1}}(h_{ij}^{k^*})^2-\frac{1}{4}\tilde{c}\sum_{\substack{i,j,k\\ j^*\neq n+1}}h_{ij}^{j^*}h_{ki}^{k^*}. \qquad (26)$$

$Q(x)$表示M在其一点x处的Ricci曲率的下确界,由式(11),对于$\alpha\neq n+1$,有

$$Q\leqslant R(X_i,X_i)=\frac{1}{4}(n-1)\tilde{c}+nH^2-H^2-\sum_{j,\beta\neq n+1}(h_{ij}^{\beta})^2=\frac{1}{4}(n-1)\tilde{c}+nH^2-H^2-(h_{ii}^{\alpha})^2-\sum_{j,\beta\neq\alpha,n+1}(h_{ij}^{\beta})^2, \qquad (27)$$

由式(27)得

$$\sum_{i,j,\beta\neq n+1}(h_{ij}^{\beta})^2\leqslant\frac{1}{4}n(n-1)\tilde{c}+n^2H^2-nH^2-nQ. \qquad (28)$$

$$\sum_i(h_{ii}^{\alpha})^4\geqslant\frac{1}{n}\Big[\sum_i(h_{ii}^{\alpha})^2\Big]^2=\frac{1}{n}(\mathrm{tr}H_{\alpha}^2)^2. \qquad (29)$$

对于α,记h_{ii}^{α}为矩阵$\boldsymbol{H}_{\alpha}$的特征值,据文献[3]可见

$$-\sum_{\beta\neq n+1}\mathrm{tr}(\boldsymbol{H}_{\alpha}\boldsymbol{H}_{\beta}-\boldsymbol{H}_{\beta}\boldsymbol{H}_{\alpha})^2=\sum_{\substack{i,j\\ \beta\neq\alpha,n+1}}(h_{ij}^{\beta})^2(h_{ii}^{\alpha}-h_{jj}^{\alpha})^2\leqslant 2\sum_{\substack{i,j\\ \beta\neq\alpha,n+1}}(h_{ij}^{\beta})^2[(h_{ii}^{\alpha})^2+(h_{jj}^{\alpha})^2]\leqslant 4\sum_{\substack{i,j\\ \beta\neq\alpha,n+1}}(h_{ij}^{\beta})^2(h_{ii}^{\alpha})^2. \qquad (30)$$

由式(28)、(29),将式(30)变为

$$-\sum_{\beta\neq n+1}\mathrm{tr}(\boldsymbol{H}_{\alpha}\boldsymbol{H}_{\beta}-\boldsymbol{H}_{\beta}\boldsymbol{H}_{\alpha})^2\leqslant 4\sum_i\Big[\frac{1}{4}(n-1)\tilde{c}+nH^2-H^2-(h_{ii}^{\alpha})^2-Q\Big](h_{ii}^{\alpha})^2\leqslant 4\Big[\frac{1}{4}(n-1)\tilde{c}+nH^2-H^2-Q\Big]\sum_i(h_{ii}^{\alpha})^2-\frac{4}{n}(\mathrm{tr}H_{\alpha}^2)^2. \qquad (31)$$

把式(28)、(31)代入式(26)及已知条件$\tilde{c}\leqslant 0$,得

$$\frac{1}{2}\Delta\Big(\sum_{\alpha\neq n+1}(h_{ij}^{\alpha})^2\Big)\geqslant\sum_{\substack{i,j,k,\alpha\\ \alpha\neq n+1}}(h_{ijk}^{\alpha})^2+(4H^2+4Q-(n-1)\tilde{c}-4nH^2)\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h_{ij}^{\alpha})^2+\frac{4}{n}\sum_{\alpha\neq n+1}(\mathrm{tr}\boldsymbol{H}_{\alpha}^2)^2-$$

$$\sum_{\alpha\neq n+1}(\mathrm{tr}\,\boldsymbol{H}_{\alpha}^2)^2+nH^2\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h_{ij}^{\alpha})^2+\frac{1}{4}\tilde{c}\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h_{ij}^{\alpha})^2+\frac{1}{4}n\tilde{c}\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h_{ij}^{\alpha})^2\geqslant\sum_{\substack{i,j,k,\alpha\\ \alpha\neq n+1}}(h_{ijk}^{\alpha})^2+\Big[\frac{1}{4}(5-3n)\tilde{c}+$$

$$(4-3n)H^2+4Q\Big]\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h_{ij}^{\alpha})^2-\frac{n-4}{n}\Big[\frac{1}{4}n(n-1)\tilde{c}+n^2H^2-nH^2-nQ\Big]\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h_{ij}^{\alpha})^2\geqslant\Big[(2n-n^2)\Big(\frac{1}{4}\tilde{c}+H^2\Big)+nQ+\frac{1}{4}\tilde{c}\Big]\sum_{\substack{i,j,\alpha\\ \alpha\neq n+1}}(h_{ij}^{\alpha})^2. \qquad (32)$$

则当Q满足

$$Q\geqslant\frac{1}{4}(n-2)(\tilde{c}+4H^2)-\frac{1}{4n}\tilde{c}, \qquad (33)$$

M为全脐的. 证毕.

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